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while (true)
On the Fluidity of Signs in Hegel, Gödel, and Turing

Introduction

The universal machine, which the Englishman Alan Turing designed and actually constructed later, exhibits in principle a number of structural similarities to systems that the German philosopher Georg Wilhelm Friedrich Hegel began to develop 130 years earlier. By stating that their logical constructions manifest a considerable degree of closeness, I am not suggesting that Hegel anticipated the computer. Neither is there any evidence that Turing sought to imitate Hegel's system of thought. Indeed, during Turing's student days in 1930's Cambridge the intellectual climate was decidedly anti-Hegelian. In 1914, the analytical philosopher Bertrand Russell had published a refutation of Hegel, *Our Knowledge of the External World*,[1] in which he attempts to show that the idealist misapprehends the meaning of the copula "is". However, the proximity of Hegel's and Turing's systems makes it possible to locate them in a history of the mechanisation of thought as a means of understanding and to illuminate the one from the perspective of the other.

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Hegel's *Science of Logic* begins with two concepts: "being" and "nothing". Both concepts are pure symbols in the sense that they mean nothing, that is, they do not reference anything particular in the external world, but merely point to the void:

"[I]t is altogether the same as what the Indian calls Brahma, when for years on end, physically motionless and equally unmoved in sensation, conception, phan-

1 Bertrand Russell, *Our Knowledge of the External World* (Chicago and London: Open Court Publishing Co., 1914).

tasy, desire and so on, looking only at the tip of his nose, he says inwardly only *Om*, *Om*, *Om*, or else nothing at all."

These symbols are so general that they even precede the differentiation into letters and numerals and can be named at will: "With this […] indeterminateness and vacuity of conception, it is indifferent whether this abstraction is called space, pure intuiting, or pure thinking."[2] The pre-literacy can also be recognised from the circumstance that the nothingness of being also emerges when one does not speak of it or write it down, but simply when it is shown, as at the beginning of Hegel's *Phenomenology of Spirit*: the Now "has already ceased to be in the act of being pointing to it".[3] Decisive for further progress is only that the signs are distinct and thereby mark the difference between them.

The first two symbols of Hegel's system no longer reference the external world, also because the adequation theory of truth had failed, at the latest in Immanuel Kant's theory of the "thing in itself".[4] Instead of comparing words with things, as adequation theory does, Hegel chooses an approach that is purely internal to consciousness and compares only concepts: "Consciousness provides its own criterion from within itself, so that the investigation becomes a comparison of consciousness with itself."[5] The second reason for the symbols' emptiness is that they are at the beginning of Hegel's system and, therefore, have to be undetermined and unmediated. Any specific content would contradict this because, being mediated, something would precede it.[6]

During Hegel's lifetime, the way was cleared for a further breakaway of the symbolic from the real world. In 1829, the Russian mathematician Nikolai Ivanovich Lobachevsky publishes an essay on hyperbolic geometry, which later results in a fundamental crisis in mathematics.[7] Interest in non-Euclidean spaces

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2 Georg Wilhelm Friedrich Hegel [1812–1831], *Science of Logic*, trans. A.V. Miller (London, NJ: Allen & Unwin, Humanities Press, 1990), p. 97. "OMOMOM" looks digital and can be interpreted as "42" ("101010"); see Douglas Adams, *The Hitchhiker's Guide To The Galaxy* (London: Pan Books, 1979).

3 G.W. F. Hegel [1807], *Phenomenology of Spirit*, trans. A.V. Miller (Oxford: Clarendon Press, 1979), p. 63.

4 Cf. Immanuel Kant [1781–1787], *Critique of Pure Reason*, trans. P. Guyer (Cambridge: Cambridge University Press, 1998), p. 185, B 59: "what may be the case with objects in themselves and abstracted from all this receptivity of our sensibility remains entirely unknown to us."

5 Hegel, *Phenomenology*, p. 53.

6 Cf. Hegel, *Logic*, p. 67f.: "With What Must the Science Begin?"

7 Nikolai Ivanovich Lobachevsky, The fundaments of geometry. *Kazanskii Vestnik* 25, 27, and 28 (1829), in Russian. Both sources and content of this paper are very unclear. An alternative first publiincreases through the publications by Bernhard Riemann and Felix Klein, who marries a grand-daughter of Hegel's in 1875.[8] Simple intuitive basic assumptions, such as Euclid's fifth axiom, which states that non-parallel straight lines extended indefinitely cross just once, are demonstrated to be false when applied to spherical surfaces.[9] In 1895, Felix Klein brings David Hilbert from Königsberg into his research team in Göttingen. According to Hilbert, it is necessary to abandon all reference to the real world through counting and measuring and, instead, establish geometry as an abstract system of symbols that does without intuitive and illustrative assumptions with the aim of giving metamathematical proofs of its consistency: "one must always be able to say, instead of 'points, straight lines, and planes', 'tables, chairs, and beer mugs'".[10] The mathematicians decide to uncouple their discourse completely from external reality and ground the world of numbers entirely within itself. Although Hilbert's words do not appear to forswear the world of the senses, this step is based upon the distrust of sensory perception, which is seen as deceptive, a widespread view that has prevailed since classical antiquity.[11] From this point in time, numbers are treated as "any system of things"[12], without reference to the world and ignoring their

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cation is: Janos Bolyai, Appendix, scientiam spatii absolute veram exhibens a veritate aut falsitate Axiomatis XI Euclidei (a priori haud unquam decidenda) independentem: adjecta ad casum falsitatis, quadratura circuli geometrica. In: Farkas Bolyai, *Tentamen in elementa matheseos purae, elementaris ac sublimioris, methodo intuitiva, evidentiaque huic propria, introducendi. Cum appendice triplici* (Marosvásárhely, 1832).

8 Bernhard Riemann [1854], *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen.* Habilitationsschrift. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, vol. 13 (1868); Felix Klein, Über die sogenannte nicht-euklidische Geometrie. *Nachrichten von der Königl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen* 17 (1871): 419–433.

9 Cf. Euclid [ca. 300 B.C.], *Elements*, trans. I. Todhunter (London: Dent, 1933), p. 6: "if a straight line meets two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines, being continually produced, shall at length meet on that side on which are the angles which are less than the two right angles."

10 Otto Blumenthal [1935], David Hilberts Lebensgeschichte. In: David Hilbert, *Gesammelte Abhandlungen*, vol. 3 (New York: Chelsea, 1965), pp. 388–429, here p. 403. Hilbert is said to have made this remark in 1891 on the way home from a lecture by Hermann Wiener.

11 Cf. Plato [ca. 387 B.C.], *Phaedo*, trans. R. Hackforth (London: Cambridge University Press, 1972), p. 83: "Now were we not saying some time ago that when the soul makes use of the body to investigate something [...] it is dragged by the body towards objects that are never constant, and itself wanders in a sort of dizzy drunken confusion, inasmuch as it is apprehending confused objects?"

12 David Hilbert [1928], Problems of the grounding of mathematics. In: Paolo Mancosu, *From Brouwer to Hilbert: The Debate on the Foundations of Mathematics in the 1920s* (New York: Oxford University Press, 1998), pp. 227–233, here p. 232.

ordering nature. Being formalistic, mathematics is a game with empty symbols, which offer as little to apprehend or contemplate as "being".

Similarly, Turing's concept of a universal machine, as described in his essay "On computable numbers", is part of this tradition and utilises symbols that precede the distinction between letters and numbers, reference nothing, and are completely meaningless. The essay text is itself a babylonian mixture of indifferent signs. In addition to Arabic numbers and Roman capital and small letters, Turing uses Fraktur capitals and small letters, symbols from predicate calculus (∂), Roman capital letters in script, and Greek capitals and small letters.[13] In the descriptive language of the essay, these symbols serve to differentiate between various abstract entities ("classes"). In the symbol set of the machine he describes, like "being" and "nothing", they mark the pure difference, to which meaning can only be ascribed subsequently and arbitrarily. If the machine's tape is inscribed with OMOMOM and the programme transforms this into OOMOOOMMOMM, at first it cannot be decided whether the machine has squared 42 or composed a new mantra. Turing machines can thus be constructed from almost any kind of material, such as DNA, mirrors, model railways, or hosepipes.[14] The Manchester "Baby" computer, which was completed in 1948 with the participation of Alan Turing, worked with an alphabet in base 32. Its first symbol was neither a letter nor a number but a forwardslash, /. "The result was that pages of programs were covered with strokes – an effect which at Cambridge was said to reflect the Manchester rain lashing at the windows."[15] Memory and processor were cathode ray tubes, that is, television screens; thus the workings and outputs of the machine were seen as a "mad dance" of flickering dots on the screen, which up to the final breakthrough on 21 June 1948 was each time a "dance of death".[16] Particularly the fact that the symbols are not letters or numbers endows the ideas of Turing and Gödel with their fundamental power.

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13 Alan M. Turing, On computable numbers, with an application to the Entscheidungsproblem. *Proc. London Math. Soc.* 42/2 (1937): 230–265.

14 See Y. Benenson, T. Paz-Elizur, R. Adar, E. Keinan, Z. Livneh, and E. Shapiro, Programmable and autonomous computing machine made of biomolecules. *Nature* 414 (2001): 430–434; Christopher Moore, Predictability and undecidability in dynamical systems. *Physical Review Letters* 64 (1990): 2354–2357; Severin Hofmann and David Moises, Turing Train Terminal. 2003/4. http://www.monochrom.at/turingtrainterminal/pictures_eng.htm [accessed 4 March 2005]; Paulo Blikstein, *Programmable Water* (Cambridge, MA: MIT, 2003). http://web.media.mit.edu/~paulo/courses/howmake/mlfabfinalproject.htm [accessed 4 March 2005].

15 Andrew Hodges, *Alan Turing: The Enigma* (New York: Simon & Schuster, 1983), p. 399.

16 Ibid., p. 392; cf. Fig. 1.

Their ideas do not concern specific symbols but symbols in general. This also avoids assigning to the computer a one-sided bias: either to the field of letters or to the field of numbers. A Turing machine only processes that which stands at the beginning of Hegel's *Logic*: pure difference. The only condition for the symbols is that they are distinguishable; that is why their number is finite.[17]

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Because the sign "being" has no content, it passes over into "nothing". The difference between the two is merely supposed. They are not different but should only be distinguished. "Nothing" is an empty symbol and, therefore, the same as "being". Both symbols are abstract and meaningless: "*Pure being* and *pure nothing* are, therefore, the same."[18] Hegel writes that this "thesis" is "so paradoxical", "indeed […] one of the hardest tasks thought poses for itself".[19] There are only two instances of the word "paradox", or "paradoxical", in the whole of Hegel's three main works. In paragraph 104 of the *Encyclopaedia*, Pythagoras' basic determination of things as numbers is referred to as such, and in connection with this, in paragraph 301, phenomena of the objective appearance of harmony, for example, that one string can produce several notes, several strings just one note, or two strings a third note, and so on. Paradoxically, Hegel uses the word very seldom because it is fundamental to his theory and is throughout denoted by the concept of "dialectic". Hegel points out that, in general, we distinguish between things on the basis of some common ground, for example, between two species of the same genus: "In contrast, with Being and Nothing the difference is in its bottomlessness and, therefore, is none, since both determinations are the same bottomlessness."[20] The difference between the two symbols breaks down because they are both empty and, therefore, the same. At the same time, the difference continues to exist and for this reason the sentence is paradoxical, as are all asser-

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17 Turing, On computable numbers, p. 249: "If we were to allow an infinity of symbols, then there would be symbols differing to an arbitrarily small extent."

18 Hegel, *Logic*, p. 82.

19 G.W.F. Hegel [1830], *Encyclopedia of the Philosophical Sciences in Outline and Critical Writings*, trans. E. Behler (New York: Continuum, 1990), vol. I, p. 70.

20 "Beim Sein und Nichts dagegen ist der Unterschied in seiner Bodenlosigkeit, und eben darum ist es keiner, denn beide Bestimmungen sind dieselbe Bodenlosigkeit." Translation, D. L. (G.W. F. Hegel [1830], *Enzyklopädie der philosophischen Wissenschaften. Werke 8, 9, 10* (Frankfurt am Main: Suhrkamp, 1970), vol. 8, p. 187).

tions of identity. The sentence distinguishes between the two symbols and relates them to each other at the same time. It represents the identity of identity and non-identity, and is paradigmatic of speculative thought:

"So, too, in the philosophical proposition the identification of Subject and Predicate is not meant to destroy the difference between them, which the form of the proposition expresses; their unity, rather, is meant to emerge as a harmony."[21]

The sentence does not consist in the circumstance that attributes are ascribed to a subject, but that the signs change into each other in the "movement of the Notion", like two vibrating strings that produce a third note.[22] The "fixed thoughts" are transformed into "a fluid state" and set in motion.[23]

In 1899, Hilbert proved the consistency of Euclidean geometry under the supposition that the theory of real numbers is consistent. In 1872, Richard Dedekind had succeeded in deriving the real numbers from whole numbers. Thus the question was then, is the theory of whole numbers consistent?[24] At the International Congress of Mathematicians in 1900 in Paris, Hilbert presented a list of tasks for the coming century in the form of 23 unsolved mathematical problems. Problem No. 2 concerned the freedom from contradiction of the arithmetical axioms.[25] Gottlob Frege attempted to solve this with a further development of Cantor's set theory, which avoids reference to the ordering character of numbers. However, in 1901, the 30-year-old Bertrand Russell came upon the very paradoxical concept of the set of all sets, which do not contain themselves. This contains itself, when it does not contain itself and does not contain itself when it contains itself. In contrast to the Cretan paradox, it is con-

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21 Hegel, *Phenomenology*, p. 38.

22 "It is the same with the other case, where, when following Tartini two different strings of a guitar are strummed, the wonderful happens, that apart from their sound a third sound is heard that, however, is not a mere mixing of the first two, not only an abstract neutral." ("Ebenso ist es dann auch mit dem anderen Fall, wo, wenn man nach Tartini, zwei verschiedene Saiten einer Gitarre anschlägt, das Wunderbare geschieht, daß man außer ihren Tönen auch noch einen dritten Ton hört, der aber nicht bloß die Vermischung der beiden ersten, kein bloß abstrakt Neutrales ist.") Translation, D.L. (Hegel, *Enzyklopädie*, vol. 9, p. 183).

23 Hegel, *Phenomenology*, p. 20.

24 David Hilbert [1899], *Grundlagen der Geometrie* (Stuttgart: Teubner, 1968); Richard Dedekind, *Stetigkeit und irrationale Zahlen* (Braunschweig: Vieweg, 1872).

25 David Hilbert, Mathematical problems. *Bulletin of the American Mathematical Society* 8 (1902): 437–479, here p. 447: "But above all I wish to designate the following as the most important among the numerous questions which can be asked with regard to the [arithmetical, D.L.] axioms: *To prove that they are not contradictory, that is, that a finite number of logical steps based upon them can never lead to contradictory results.*"

clusive.[26] In a letter dated 16 June 1902, Russell communicated his discovery to the 54-year-old Frege, who was just preparing the second edition of his book, *Grundsätze der Arithmetik*.[27] Frege's reply was humble – and shocked: "Your discovery of the contradiction has surprised me beyond words and, I should almost like to say, left me thunderstruck, because it has rocked the ground on which I meant to build arithmetic."[28] Frege added a resigned afterword to the second volume of his book and, after this incident, gave up set theory.

In Hegel, thought was driven to "nothing", when it attempted to hold fast to "being", and to "being", when it attempted to hold fast to "nothing", moving eternally in circles. Also here, thought gets into a giddy circular motion that, looking back, Russell describes as follows: "giving a person a piece of paper on which is written: 'The statement on the other side of this paper is false'. The person turns the paper over, and finds on the other side: 'The statement on the other side of this paper is true'." Russell was convinced that the origin of such paradoxes lay in the self-application of statements, and over the next ten years he developed a theory of types, together with his teacher, Alfred North Whitehead, which attempted to prevent such mixing – the *Principia Mathematica.* The profundity, lengthiness, and difficulty of their undertaking can be gauged by the fact that 1 + 1 = 2 is only proved on page 379.[29]

These flickering sentences, which continually drive thinking in circles are dangerous because they annul mathematics as a decision-making procedure between true and false and push it into a realm of false and total truth. Hodges writes of the problem of freedom from contradiction:

"And that spelt disaster. In any purely logical system there was no room for a single inconsistency. If one could ever arrive at '2 + 2 = 5' then it would follow that '4 = 5' and '0 = 1', so that any number was equal to 0, and so that every proposition whatever was equivalent '0 = 0' and therefore true."[30]

The system would be supra-universal and would lose all possibility of of dis-

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26 In its usual form, the Cretan paradox is not conclusive because the opposite of "all Cretans lie" in predicate logic is "one Cretan tells the truth". Thus the Cretan who speaks is lying and some other Cretan tells the truth.

27 Cf. Jean van Heijenoort, *From Frege to Gödel. A Source Book in Mathematics, 1879–1931* (Harvard: Harvard University Press, 1967), p. 124f.; Hodges, *Enigma*, p. 84.

28 Heijenoort, *Source Book*, p. 127f.

29 Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, 3 vols. (Cambridge: Cambridge University Press, 1910, 1912, 1913), vol. 1, p. 379; cf. Fig. 2.

30 Hodges, *Enigma*, p. 84; the emphasis is mine – D.L.

SECTION A] CARDINAL COUPLES 379

$*54{\cdot}42.\ \vdash :: \alpha \epsilon 2 \,.\supset :. \beta \subset \alpha \,.\, \exists ! \beta \,.\, \beta \neq \alpha \,.\equiv .\, \beta \epsilon \iota``\alpha$

Dem.

$\vdash .\, *54{\cdot}4 \,.\quad \supset \vdash :: \alpha = \iota`x \cup \iota`y \,.\supset :.$

$\beta \subset \alpha \,.\, \exists ! \beta \,.\equiv : \beta = \Lambda \,.\vee .\, \beta = \iota`x \,.\vee .\, \beta = \iota`y \,.\vee .\, \beta = \alpha : \exists ! \beta :$

$[*24{\cdot}53{\cdot}56 . *51{\cdot}161] \quad \equiv : \beta = \iota`x \,.\vee .\, \beta = \iota`y \,.\vee .\, \beta = \alpha$ (1)

$\vdash .\, *54{\cdot}25 \,.\, \text{Transp} \,.\, *52{\cdot}22 \,.\supset \vdash : x \neq y \,.\supset .\, \iota`x \cup \iota`y \neq \iota`x \,.\, \iota`x \cup \iota`y \neq \iota`y :$

$[*13{\cdot}12] \quad \supset \vdash : \alpha = \iota`x \cup \iota`y \,.\, x \neq y \,.\supset .\, \alpha \neq \iota`x \,.\, \alpha \neq \iota`y$ (2)

$\vdash .\, (1) \,.\, (2) \,.\supset \vdash :: \alpha = \iota`x \cup \iota`y \,.\, x \neq y \,.\supset :.$

$\beta \subset \alpha \,.\, \exists ! \beta \,.\, \beta \neq \alpha \,.\equiv : \beta = \iota`x \,.\vee .\, \beta = \iota`y :$

$[*51{\cdot}235] \quad \equiv : (\exists z) \,.\, z \epsilon \alpha \,.\, \beta = \iota`z :$

$[*37{\cdot}6] \quad \equiv : \beta \epsilon \iota``\alpha$ (3)

$\vdash .\, (3) \,.\, *11{\cdot}11{\cdot}35 \,.\, *54{\cdot}101 \,.\supset \vdash .\, \text{Prop}$

$*54{\cdot}43.\ \vdash :.\, \alpha, \beta \epsilon 1 \,.\supset : \alpha \cap \beta = \Lambda \,.\equiv .\, \alpha \cup \beta \epsilon 2$

Dem.

$\vdash .\, *54{\cdot}26 \,.\supset \vdash :.\, \alpha = \iota`x \,.\, \beta = \iota`y \,.\supset : \alpha \cup \beta \epsilon 2 \,.\equiv .\, x \neq y \,.$

$[*51{\cdot}231] \quad \equiv .\, \iota`x \cap \iota`y = \Lambda \,.$

$[*13{\cdot}12] \quad \equiv .\, \alpha \cap \beta = \Lambda$ (1)

$\vdash .\, (1) \,.\, *11{\cdot}11{\cdot}35 \,.\supset$

$\vdash :.\, (\exists x, y) \,.\, \alpha = \iota`x \,.\, \beta = \iota`y \,.\supset : \alpha \cup \beta \epsilon 2 \,.\equiv .\, \alpha \cap \beta = \Lambda$ (2)

$\vdash .\, (2) \,.\, *11{\cdot}54 \,.\, *52{\cdot}1 \,.\supset \vdash .\, \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Alfred North Whitehead and Bertrand Russell, Principia Mathematica, *3 vols. (Cambridge: Cambridge University Press, 1910, 1912, 1913), vol. 1, p. 379, note 29.*

tinguishing between true and false. To illustrate the term "supra-universal", I cite a statement by the American Defence Minister Donald Rumsfeld, which interprets the bestial treatment of Iraqi prisoners in Abu-Ghraib prison as a positive sign: "The system worked."[31]

The crisis in mathematics is complete when, in 1931, Kurt Gödel proves that all formal systems, like the *Principia Mathematica*, must contain of necessity undecidable propositions and, in contrast to reason, cannot achieve certain true propositions. In 1928, Hilbert again formulated his project in three questions: Is

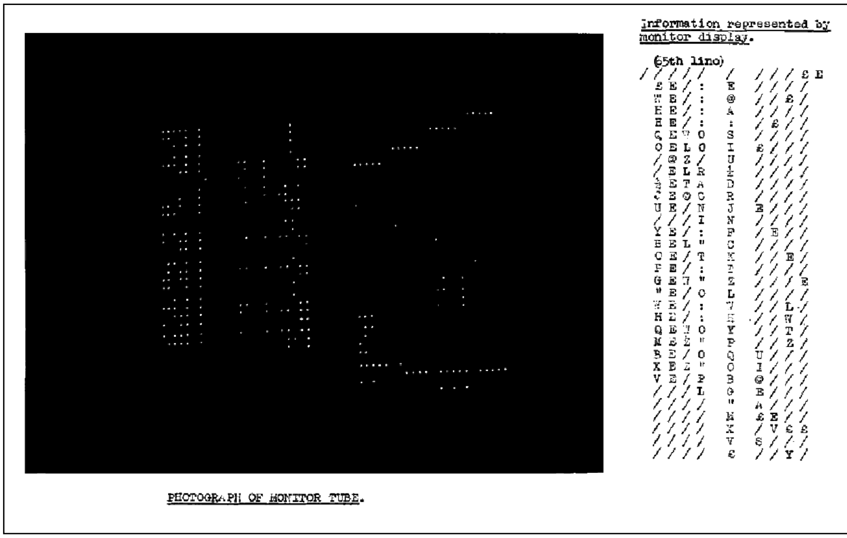
31 Donald Rumsfeld, NBC Interview, 5 May 2004. http://www.pentagon.mil/transcripts/2004/tr20040505-secdef1425.html [accessed 28 February 2005].

mathematics complete? Is it free of contradictions? Is it decidable? With Gödel's discovery, Hilbert's project has failed. Whereas Russell had still assumed that self-application generates paradoxes from time to time, Gödel demonstrated that paradoxes are a necessary part of all sufficiently complex formal systems that cannot be eliminated. Hegel formulates this in an even more radical fashion when he states "that every actual thing involves a coexistence of opposed elements. Consequently, to know, or, in other words, to comprehend an object is equivalent to being conscious of it as a concrete unity of opposed determinations".[32] The fundament in reality that had been lost with the advent of non-Euclidean geometry cannot be recovered through formalistic attempts to ground mathematics.

The catastrophe of the paradox that it is not possible to distinguish between "being" and "nothing", "true" and "false", or "0" and "1" and that one passes into the other in circles, is resolved in a fruitful manner by Alan Turing. The infinite transformation of symbols that have become meaningless into one another is the modus operandi of the Turing machine. If the machine is in a state "0" when the symbol "0" is encountered and the command "1R1" is executed, this happens twice. Both in the machine's state memory and on the paper tape "0" changes into "1". To program an addition, exactly one "0" (the one between the blocks of summands) must be transformed into "1" and one "1" into "0". Thus the chain of symbols "OOOMOO" (3 + 2) becomes "OOOOOM", a result that can be interpreted as 5. Instead of laying down rules of transformation for how one true statement can be derived from another, as in formalist mathematics, Turing constructs a general machine for transforming symbols. From this point onwards, the truth or falseness of the transformation lies in the hands of the software developer. For this reason, the very first programmers' manual, Turing's *Programmer's Handbook for Manchester Electronic Computer Mark II*, devotes much time to specifying how errors in the software can be detected and rectified.[33] The Turing machine implements

32 "[…] alles Wirkliche entgegengesetzte Bestimmungen in sich enthält und daß somit das Erkennen und näher das Begreifen eines Gegenstandes eben nur so viel heißt, sich dessen als einer konkreten Einheit entgegengesetzter Bestimmungen bewußt zu werden." Translation, D.L. (Hegel, *Enzyklopädie*, vol. 8, p. 128).

33 Alan M. Turing, *Programmers' Handbook for Manchester Electronic Computer Mark II.* (Manchester, 1951). http://www.alanturing.net/turing_archive/archive/m/m01/M01-001.html [accessed 3 March 2005], p. 59: "Programming is a skill best acquired by practice and example rather than from books. The remarks here are therefore quite inadequate." Notwithstanding, Turing proceeds to elaborate this theme on the following 24 pages.



http://www.alanturing.net/turing_archive/archive/m/m01/M01-001.html

the identity of identity and difference directly, namely, technically. Empty symbols, which exhibit a bottomless difference and pass over into each other, are also its foundation.

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In formalist mathematics not only are letters and numbers the same thing, namely symbols, but in the generality the difference between numbers, operations with them, and statements about them vanishes. Gödel and Turing are able to answer Hilbert's questions in the negative because they make functions and numbers into one and thus have the possibility to encode metamathematical statements as numbers and apply them to themselves. Logical symbols, such as disjunction, negation, and generalisation are translated simply into natural numbers at the beginning of Gödel's essay:

“0” … 1	“v” … 7	“(“ … 11
“*f*” … 3	“Π” … 9	“)” … 13
“~” … 5		

"Naturally, for metamathematical considerations, it makes no difference which objects one takes as primitive symbols, and we decide to use natural numbers for that purpose."[34] By means of a system of prime number exponents, as already used by Leibniz, Gödel prevents the collision of operators coded in numbers and real natural numbers.[35]

Put simply, the Turing machine, too, consists in one chain of symbols, which represents the data computed – the symbols on the tape – and another chain, which specifies the operations – the programme. However, the particular dynamics and universality arises from the circumstance that the symbols on the tape also determine the way the programme runs. They are instructions in the form of markers and data at the same time. Only through this can a general routine be written that adds two numbers. Further, each particular Turing machine is a chain of symbols that feeds into the universal machine.

Similarly, in Hegel the dialectic is first set in motion when the meaning of the empty symbol "being" is thematised by further symbols. Only in the blend of meta- and object-language can it be established that "being" means "nothing". It is "natural", because "'Notion' and 'object' […] both fall *within* that knowledge which we are investigating".[36] Hegel depicts immanent a self-critical mind, which makes a concept like "being" from the world and always turns back to it again to check whether the concept coincides with what he means. Each time the realisation of a concept's limitedness forces the mind to go on to a new concept, such as "becoming", which contains the identity of "being" and "nothing" as aspects and is reflected again. As in the approaches of Gödel and Turing, it is the permanent self-application of statements that produces paradoxes that do not allow the mind to stop turning.

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34 Kurt Gödel [1931], On formally undecidable propositions of *Principia Mathematica* and related systems. In: *The Undecidable. Basic Papers on Undecidable Propositions, Unsolvable Problems, and Computable Functions*, ed. Martin Davis (Hewlett, NY: Raven, 1965), pp. 4–38, here pp. 7 and 13; the emphasis is mine – D.L.

35 On Leibniz, see Gerhard F. Strasser, *Lingua Universalis. Kryptologie und Theorie der Universalsprachen im 16. und 17. Jahrhundert* (Wiesbaden: Harrassowitz, 1988), p. 241.

36 Hegel, *Phenomenology*, p. 53.

At the system's end, it bends back to its beginning and thus forms a cycle of endless becoming. The "absolute idea", the highest concept in Hegel's *Science of Logic* ultimately ends by transforming into "being": "[T]he science exhibits itself as a *circle* returning upon itself, the end being wound back into the beginning, the simple ground, by the mediation; this circle is moreover a *circle of circles*."[37] This is necessary in order to motivate subsequently the at first undeterminate and groundless beginning. A deductively progressing system with a claim to universality must, at its end, when it has deduced the totality of all facts, turn back to its beginning because its beginning is the only thing that has not yet been deduced. Also symbolic spaces, which achieve universality through recombination, like Jorge Luis Borges' *Library of Babel*, which is "total" and whose "shelves register all the possible combinations of the twenty-odd orthographical symbols […]: in other words, all that it is given to express, in all languages" also turn into their beginning at their end: "*The Library is unlimited and cyclical.* If an eternal traveller were to cross it in any direction, after centuries he would see that the same volumes were repeated in the same disorder."[38] The founding father of combinatory systems, Raimundus Lullus, used circular disks to set his text machine in motion.[39]

Turing's construction is also an endless iterative loop. Contrary to popular belief, the programmes proposed in *On Computable Numbers* never stop. The text distinguishes between "circular" and "circle-less" ones. The circular programmes specify the computed real number endlessly by giving it further digits through sub-routines. The circle-less programmes reach "a configuration from which there is no possible move", or run on but do not print out any further numerical symbols.[40] Algorithms, which stop and deliver a result, do not occur in the first design of the universal machine, only algorithms that fail or remain in a state of becoming, endlessly modifying the result.

The majority of today's computer programmes are also designed to run endlessly. To this end, the software developer encloses the core of the algorithm in a so-called run-loop, an iterative loop, which is executed under the tautological

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37 Hegel, *Logic*, p. 842.

38 Jorge Luis Borges [1944], *Labyrinths. Selected Stories and Other Writings*, trans. D. Yates (Harmondsworth, Middlesex: Penguin Books, 1979), pp. 81 and 85.

39 See Amador Vega, *Ramon Llull and the Secret of Life* (New York: Herder & Herder, 2003), p. 62f.

40 Turing, *Numbers*, p. 233.

condition "while(true)" or "while(1)". It is precisely this basis, that true remains true and 1=1, that was shattered by Gödel's developments. If one were to formalise the actual basis of software, it would be: while(0 != 1 & 0 == 1). "while(true)" secures the algorithm against its own bottomlessness. It still reveals an echo of the shock triggered by Gödel.

stop()

Hegel distinguishes between two forms of the identity of identity and difference of "being" and "nothing", namely, "becoming" ["Werden"] and "determinate being" ["Dasein"]. In the former, "being" and "nothing" are only present in the form of vanishing into each other, origination and passing. Becoming, however, must "vanish also. Becoming is as if it were a fire, which dies out in itself, when it consumes its material." The further negation of "becoming" and the result of this process is "determinate being", definite being, and thus different from other being – this is the side of negation.[41]

The first occurrence of "determinate being" in computer science is in an article, only three pages long, by the mathematician Emil Post. Post developed strikingly similar ideas to Turing, at around the same period and entirely independently. His basic model is not like Turing's – an idealised typewriter – but instead, a production line worker, who processes a series of boxes according to instructions, which he can again mark with a forwardslash, /. Post distinguishes between three types of commands: the first is independent of any mark, the second is a case differentiation, which commands this or that depending on whether there is a forwardslash or not, and the third command is "stop".[42] For Post, a programme is only considered as the solution of a problem when the process, which the programme sets in motion, stops for each specific input, that is, it reaches the third command.

In a similar way, in Tibor Rado's much later reformulation of the Turing machine the property of circle-lessness is interpreted in such a way that the programmes reach the so-called stop sign and leave a certain result on the tape,

41 Hegel, *Logic*, p. 105f.; "[…] so ist es [das Dasein, D.L.] selbst ein Verschwindendes, ein Feuer gleichsam, welches in sich selbst erlischt, indem es sein Material verzehrt." Translation, D. L. (Hegel, *Enzyklopädie*, vol. 8, p. 195).

42 Emil Post [1936], Finite combinatory processes. Formulation I. In: Davis, *Undecidable*, pp. 288–291, here p. 290.

whereas circular programmes modify the symbols in a state of eternal becoming.[43] The modification of Turing's approach, which continues to exert influence today, is indebted to Kleene's authoritative text *Introduction to Metamathematics* of 1952: "Our treatment here is closer in some respects to Post 1936."[44]

"Becoming is an unstable unrest which settles into a stable result."[45] One difference between Hegel and Turing is that in Hegel's system, reflection upon the limitedness of the empty abstract concepts increasingly fills them and they become more concrete. On the other hand, on the machine's tape – when it stops at all – there is only a chain of still empty symbols. The device can go back to what it has written, but it cannot reflect upon it, and therefore cannot move it to a higher level ["aufheben"]. Because the machine does not understand what it is doing, it can only achieve the level of simple "determinate being", provided it does not remain in the state of eternal becoming. Even though the heights of content of the absolute idea, which are attained in Hegel's reading and writing, remain closed to the machine, it is still the case, as Alan Turing wrote 50 years ago, that if "the brain work[s] in *some* definite way", it can be emulated by the universal machine.[46]

With the construction of his machine, Turing reveals the uncanny finding that the basis of mathematical operations, of which people had hoped and expected the foundation of all sciences, lacks calculability even in the simplest operations, if it is not sought in reality but in the formal processing of any "system of things". There is no general procedure to determine whether a programme is circular or circle-less, whether it will ever manage to inscribe "1" on paper, or how high the maximum result will be. In Tibor Rado's "Busy Beaver game", where the goal is to programme a *n*-state computer to write as many symbols as possible on a tape and then stop, competitors must inform the jury how many operations the entered program will make before it shuts down; otherwise, the jury cannot judge the algorithm.[47]

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43 Cf. Tibor Rado, On non-computable functions. *The Bell System Technical Journal* XLI (1962): 877–884, here p. 877f.: "The last column […] contains the index of the next card to be used, or 0 (zero), where 0 is the code for 'Stop'."

44 Stephen Cole Kleene, *Introduction to Metamathematics* (Amsterdam: North-Holland, 1952), p. 361.

45 Hegel, *Logic*, p. 106.

46 Hodges, *Enigma*, p. 420. Here Turing was obviously not thinking of the simulation of neurons: it is possible, he wrote, "to copy the behaviour of nerves, but there seems very little point in doing so. It would be rather like putting a lot of work into cars which walked on legs instead of continuing to use wheels." (Hodges, *Enigma*, p. 404).

To answer the universal claim of Hilbert's theory of proof, Turing must develop a system that is able to formulate all possible data and operations. Otherwise, there is a danger that a further extension of the machine would in fact allow the *Entscheidungsproblem* to be solved. His counter-proof is influenced by universalism and outdoes it, in that although he demonstrates that there can be no general procedure to distinguish between true and false, he constructs a formalism that can express all true and false propositions and methods of contemporary and future systems of axioms. For this reason it is pointless to think beyond the Turing machine or, indeed, beyond the binary.

The various positions, for example, of Hegel's *Phenomenology of Spirit*, are also universal. With the unreflected concepts of "I", "this", "now", and "here", sensory certainty has developed a complete understanding of the world, for thus everything is either "I" or "this".[48] This generality makes it possible to refute sensory certainty using certain examples, such as writing down the "now" of the night that will have passed.[49]

Hegel's system, too, is "supra-universal" in the sense that he develops the supposed universal truth of a position in such a way that its falsity and finiteness is demonstrated, which forces its negation in the form of a new truth. The concepts are in a state of perpetual transformation. They are not being or nothing, but becoming: an endless transition between true and false and true. Also Hegel's system claims to run through all possible, that is, also all future positions. The difficulty of answering this prophecy is reflected by Michel Foucault almost 170 years later, on the occasion of his inaugural lecture at the Collège de France:

"[O]ur entire epoch, whether in logic or epistemology, whether in Marx or Nietzsche, is trying to escape from Hegel […] But to make a real escape from Hegel presupposes an exact appreciation of what it costs to detach ourselves from him. It presupposes a knowledge of how close Hegel has come to us, perhaps insidiously. It presupposes a knowledge of what is still Hegelian in that which allows us to think against Hegel; and an ability to gauge how much our

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47 Rado, *Functions*, p. 879: "Beyond the enormous number of cases to survey, he will find that it is very hard to see whether certain entries do stop at all. This is the reason for the requirement that each contestant must submit the shift number *s* with his entry."

48 Cf. titles of esoteric literature, such as Chiara Lubich, *Here and Now. Meditations in Living in the Present* (Hide Park, NY: New City Press, 2000); or Ram Dass, *Be Here Now* (Three Rivers Press, 1971).

49 Hegel, *Phenomenology*, p. 60f.

resources against him are perhaps still a ruse which he is using against us, and at the end of which he is waiting for us, immobile and elsewhere."[50]

Paradoxically, both systems point to something beyond themselves. In Hegel the finiteness of all positions that are gone through indicates an "absolute spirit" which is realised in the finiteness of nature and history and ultimately returns to itself.[51] This explains the words at the beginning of the *Science of Logic*: this work represents God "as he is in his eternal essence before the creation of nature and a finite mind".[52] Mighty words indeed. Infinity does not oppose finiteness, but contains it as the "wealth of the particular".[53]

Gödel and Turing construct a universal determinist system in order to demonstrate with the metaphor of paradox that freedom is conceivable within it. Already in 1928, Turing's teacher Godfrey Harold Hardy contradicted emphatically Hilbert's question as to decidability:

"There is of course no such theorem, and this is very fortunate, since if there were we should have a mechanical set of rules for the solution of all mathematical problems, and our activities as mathematicians would come to an end. […] It is only the very unsophisticated outsider who imagines that mathematicians make discoveries by turning the handle of some miraculous machine."[54]

However, proof is only forthcoming from the construction of this "miraculous machine" that we owe to Alan Turing.

Translated by Gloria Custance

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50 Foucault, Michel [1970], The order of discourse, in: R. Young, *Untyping the Text. A Poststructuralist Reader* (London: Routledge, 1981), pp. 48–78, here p. 74.

51 Hegel, *Phenomenology*, p. 493.

52 Hegel, *Logic*, p. 50.

53 Ibid., p. 58.

54 Hodges, *Enigma*, p. 93f.